Lossy Circular Waveguide

Introduction

In mode analysis it is usually the primary goal to find a propagation constant. This quantity is often, but not always, real valued; if the analysis involves some lossy part, such as a nonzero conductivity or an open boundary, the eigenvalue is complex. In such situations, the real and imaginary parts have separate interpretations:

- The real part is the propagation constant
- The imaginary part is the attenuation constant, measuring the damping in space

Model Definition

The mode analysis study for electromagnetic waves solves the eigenvalue problem

\[ \nabla \times (\mu^{-1} \nabla \times \mathbf{E}) - \lambda \mathbf{E} = 0 \]

where

\[ \lambda = k_0^2 \left( \varepsilon_r - \frac{j\sigma}{\omega} \right) \]

is the eigenvalue. For time-harmonic problems, the electric field for out-of-plane propagation can be written as

\[ \mathbf{E}(\mathbf{r}, t) = \text{Re}(\tilde{\mathbf{E}}(\mathbf{r})e^{j\omega t - \alpha z}) \]

where \( z \) is the known out-of-plane direction.

The spatial parameter, \( \alpha = \delta_z + j\beta = -\lambda \), can have a real part and an imaginary part. The propagation constant is equal to the imaginary part, and the real part, \( \delta_z \), represents the damping along the propagation direction.
**Variables Influenced by Mode Analysis**

The following table lists the variables that are influenced by the mode analysis in terms of the eigenvalue $\lambda$:

<table>
<thead>
<tr>
<th>NAME</th>
<th>EXPRESSION</th>
<th>CAN BE COMPLEX</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>beta</td>
<td>imag(-lambda)</td>
<td>No</td>
<td>Propagation constant</td>
</tr>
<tr>
<td>dampz</td>
<td>real(-lambda)</td>
<td>No</td>
<td>Attenuation constant</td>
</tr>
<tr>
<td>dampdB</td>
<td>20*log10(exp(1))*dampz</td>
<td>No</td>
<td>Attenuation per meter in dB</td>
</tr>
<tr>
<td>neff</td>
<td>j*lambda/k0</td>
<td>Yes</td>
<td>Effective mode index</td>
</tr>
</tbody>
</table>

This two-dimensional model finds the modes of a circular waveguide with walls made of a nonperfect conductor, which is copper in this case. The losses in the walls lead to attenuation of the propagating wave. The propagation constant $\beta$ is obtained as the imaginary part of $\alpha = -\lambda$ and the damping $\delta_z$ is obtained as the real part. Since the wave in the waveguide is attenuated in the $z$ direction as $e^{-\delta z}$, the attenuation in dB scale is calculated using the formula

$$\Delta_{dB} = 20\delta_z \log e$$

**Results and Discussion**

The eigenvalue solver returns six eigenvalues. Table 1 shows the six effective mode indices, $n_{eff}$, closest to 1, where

$$n_{eff} = \frac{j\lambda}{k_0}$$

and $k_0$ is the wavenumber in vacuum. The table also lists the propagation constant and damping in dB/m for each eigenmode.

**Table 1: Effective Mode Indices, Propagation Constants, and Attenuation.**

<table>
<thead>
<tr>
<th>Effective mode index</th>
<th>Propagation constant (1/m)</th>
<th>Attenuation (dB/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9308 - 2.2082·10^{-6}i</td>
<td>19.5071</td>
<td>4.0199·10^{-4}</td>
</tr>
<tr>
<td>0.9733 - 2.1116·10^{-6}i</td>
<td>20.3992</td>
<td>3.844·10^{-4}</td>
</tr>
<tr>
<td>0.9566 - 1.7954·10^{-6}i</td>
<td>20.0486</td>
<td>3.2684·10^{-4}</td>
</tr>
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<td>0.9566 - 1.7954·10^{-6}i</td>
<td>20.0486</td>
<td>3.2684·10^{-4}</td>
</tr>
<tr>
<td>0.9844 - 9.38·10^{-7}i</td>
<td>20.6324</td>
<td>1.7076·10^{-4}</td>
</tr>
<tr>
<td>0.9844 - 9.38·10^{-7}i</td>
<td>20.6324</td>
<td>1.7076·10^{-4}</td>
</tr>
</tbody>
</table>
The default surface plot shows the norm of the electric field for the effective mode index $0.9308 - 2.208 \cdot 10^{-6} j$. This plot is shown in Figure 1.

Figure 1: The surface plot visualizes the norm of the electric field for the effective mode index $0.9308 - 2.208 \cdot 10^{-6} j$.

Model Library path: RF_Module/Transmission_Lines_and_Waveguides/lossy_circular_waveguide

Modeling Instructions

From the File menu, choose New.

NEW

1 In the New window, click the Model Wizard button.

MODEL WIZARD

1 In the Model Wizard window, click the 2D button.
2 In the Select physics tree, select Radio Frequency>Electromagnetic Waves, Frequency Domain (emw).

3 Click the Add button.

4 Click the Study button.

5 In the tree, select Preset Studies>Mode Analysis.

6 Click the Done button.

GEOMETRY I

Circle 1

1 In the Model Builder window, under Component 1 right-click Geometry 1 and choose Circle.

2 In the Circle settings window, locate the Size and Shape section.

3 In the Radius edit field, type 0.5.

4 Click the Build All Objects button.

MATERIALS

On the Home toolbar, click Add Material.

ADD MATERIAL

1 Go to the Add Material window.

2 In the tree, select Built-In>Air.

3 In the Add material window, click Add to Component.

Air

By default the first material you add apply for all domains.

Next, specify copper as the material on the boundaries.

ADD MATERIAL

1 Go to the Add Material window.

2 In the tree, select Built-In>Copper.

3 In the Add material window, click Add to Component.

MATERIALS

Copper

1 In the Model Builder window, under Component 1>Materials click Copper.
2 In the **Material** settings window, locate the **Geometric Entity Selection** section.
3 From the **Geometric entity level** list, choose **Boundary**.
4 From the **Selection** list, choose **All boundaries**.

**ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN**

*Impedance Boundary Condition 1*
1 On the **Physics** toolbar, click **Boundaries** and choose **Impedance Boundary Condition**.
2 In the **Impedance Boundary Condition** settings window, locate the **Boundary Selection** section.
3 From the **Selection** list, choose **All boundaries**.

**STUDY 1**
Solve for the 6 effective mode indices closest to 1.

*Step 1: Mode Analysis*
1 In the **Model Builder** window, expand the **Study 1** node, then click **Step 1: Mode Analysis**.
2 In the **Mode Analysis** settings window, locate the **Study Settings** section.
3 In the **Desired number of modes** edit field, type 6.
4 On the **Home** toolbar, click **Compute**.

**RESULTS**

*Electric Field (emw)*
The default plot shows the electric field norm for the lowest mode found; compare with Figure 1.

*Derived Values*
Calculate the propagation constant and the attenuation constant (in dB) for each effective mode index.
1 On the **Results** toolbar, click **Global Evaluation**.
2 In the **Global Evaluation** settings window, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Electromagnetic Waves, Frequency Domain>Global>Propagation constant (emw.beta)**.
3 Click the **Evaluate** button.
   Compare the results with those in the second column of **Table 1**.
4 Click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Electromagnetic Waves, Frequency Domain>Global>Attenuation constant, dB (emw.dampzdB)**.

5 Click the **Evaluate** button.

Compare with the third column of **Table 1**.